Math 45 SSM 2/e 7.1 Simplifying Rational Expressions

Objectives:
1) Evaluate a rational expression
2) Determine values of variable that make value of the rational expression undefined.
3) Simplify rational expression
   - factor completely
   - divide out common factors.

Defn: A rational expression is a quotient of two polynomials $\frac{p}{q}$ where $q \neq 0$.

Generally speaking: a rational expression is a fraction with in variable in the denominator.

① Evaluate $\frac{p^2-9}{2p^2+p-10}$ for a) $p=-1$ b) $p=0$ c) $p=2$ d) $p=5/2$

Step 1: Substitute (using ( ) for negatives)

Step 2: Use order of operations to calculate.

a) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{(-1)^2-9}{2(-1)^2+(-1)-10}$

$= \frac{1-9}{2(1)-1-10}$

$= \frac{-8}{2-1-10}$

$= \frac{-8}{-9}$

$= \boxed{\frac{8}{9}}$

b) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{0^2-9}{2(0)^2+0-10}$

$= \frac{-9}{-10}$

$= \boxed{\frac{9}{10}}$

① con't

c) $\dfrac{p^2-9}{2p^2+p-10} \rightarrow \dfrac{2^2-9}{2(2)^2+2-10}$

$= \dfrac{4-9}{2\cdot 4+2-10}$

$= \dfrac{-5}{8+2-10}$

$= \dfrac{-5}{10-10}$

$= \dfrac{-5}{0}$

$=$ undefined

d) $\dfrac{p^2-9}{2p^2+p-10} \rightarrow \dfrac{(-5/2)^2-9}{2(-5/2)^2+(-5/2)-10}$

$= \dfrac{\frac{25}{4}-9}{2\left(\frac{25}{4}\right)-\frac{5}{2}-10}$

$= \dfrac{\frac{25}{4}-\frac{9\cdot 4}{1\ 4}}{\frac{50}{4}-\frac{5\cdot 2}{2\ 2}-\frac{10\cdot 4}{1\ 4}}$

$= \dfrac{\left(\frac{25-36}{4}\right)}{\left(\frac{50-10-40}{4}\right)}$

$= \dfrac{\frac{-9}{4}}{\left(\frac{0}{4}\right)}$

$= \left(\frac{-9}{4}\right) \div \left(\frac{0}{4}\right)$

$= \frac{-9}{4} \div 0$

$=$ undefined

② Review: Solve $2p^2+p-10=0$

-20, 5, -4, 1

$2p^2+5p-4p-10=0$

$p(2p+5)-2(2p+5)=0$

$(2p+5)(p-2)=0$

$2p+5=0$ $\qquad$ $p-2=0$

$2p=-5$ $\qquad$ $\boxed{p=2}$

$\boxed{p=\frac{-5}{2}}$

NOTICE: 1) $2p^2+p-10$ is the denominator of $\frac{p^2-9}{2p^2+p-10}$ in ①.

2) The solutions of $2p^2+p-10=0$ where $-\frac{5}{2}$ and 2.

3) $\frac{-5}{2}$ and 2 were the numbers in ① c) and ① d) that gave undefined results.

4) Undefined means $\div 0$, or denominator $=0$.

5) To find values of variable that make a rational expression undefined, we need denominator $=0$.

③ Find the values of x for which $\frac{2}{x+3}$ is undefined.

step 1: Write an equation by setting denominator $=0$.

$x+3=0$.

step 2: Solve the equation

$\boxed{x=-3}$

④ Find the values of x for which $\frac{8x}{x^2-2x-3}$ is undefined.

$x^2-2x-3=0$

$(x-3)(x+1)=0$

-3, $+1$, -3, -2

$\boxed{x=3,\ x=-1}$

⑤ Find the values for which $\frac{3h+2}{h^3+5h^2+4h}$ is undefined.

$h^3+5h^2+4h=0$

$h(h^2+5h+4)=0$

$h(h+4)(h+1)=0$

$\boxed{h=0,-4,-1}$

Simplify

(6) $\frac{6}{6} = \boxed{1}$

(7) $\frac{x}{x} = \boxed{1}$

(8) $\frac{x-2}{x-2} = \boxed{1}$

(9) $\frac{2x-10}{4x^2-20x}$

$= \frac{2(x-5)}{4x(x-5)}$

$= \frac{2}{4x}$

$= \boxed{\frac{1}{2x}}$

(10) $\frac{4-x^2}{2x^2-x-6}$

$= \frac{-x^2+4}{2x^2-x-6}$

$= \frac{-(x^2-4)}{2x^2-x-6}$

$= \frac{-(x+2)(x-2)}{(x-2)(2x+3)}$

$= \boxed{\frac{-(x+2)}{(2x+3)}}$

(11) $\frac{ab+3b-ac-3c}{a^2+6a+9}$

$= \frac{(a+3)(b-c)}{(a+3)(a+3)}$

$= \boxed{\frac{b-c}{a+3}}$

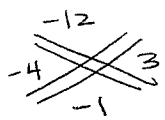
To simplify a rational expression

Step 0: Write numerator and denominator in standard form. * This is important. *

step 1: Factor everything completely. * This is essential. There is no skipping this step, no matter how close you come. *

step 2: Identify common factors in the numerator and divide (cancel) them out.

step 3: Leave final answer fully factored. (Don't FOIL or multiply.)



denominator

$2x^2-x-6$

$2x^2-4x+3x-6$

$2x(x-2)+3(x-2)$

$(x-2)(2x+3)$

numerator grouping:

$ab+3b-ac-3c$

$b(a+3)-c(a+3)$

$(a+3)(b-c)$

perfect square trinomial for denominator

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Simplify.

(12) $\dfrac{x-3}{x^3-27}$

diff of cubes
$a^3-b^3=(a-b)(a^2+ab+b^2)$
$x^3-27=(x-3)(x^2+3x+9)$

$= \dfrac{(x-3)}{(x-3)(x^2+3x+9)}$

$= \boxed{\dfrac{1}{x^2+3x+9}}$

(13) $\dfrac{x^3+64}{x^2-4x+16}$

sum of cubes
$a^3+b^3=(a+b)(a^2-ab+b^2)$
$x^3+64=(x+4)(x^2-4x+16)$

Notice! the trinomial ↑ is the same as the denominator!

$= \dfrac{(x+4)(x^2-4x+16)}{(x^2-4x+16)}$

$= \boxed{x+4}$

(14) $\dfrac{4p^2-20pq+25q^2}{6p^2-7pq-20q^2}$

numerator
perfect square trinomial
$(2p-5q)(2p-5q)$
$4p^2-10pq-10pq+25q^2$ ✓

denominator
$6p^2-7pq-20q^2$

-120
-15 8
-7

1, −120
2, −60
3, −40
4, −30
5, −24
6, −20
8, −15

$= 6p^2-15pq+8pq-20q^2$

$= 3p(2p-5q)+4q(2p-5q)$

$= (2p-5q)(3p+4q)$

$= \boxed{\dfrac{2p-5q}{3p+4q}}$

Simplify.

NO (15) $\dfrac{2z^2+6z+4}{-4z-8}$

$= \dfrac{2(z+1)(z+1)}{-4(z+2)}$

$= \dfrac{-2}{4}(z+1)$

$= \boxed{\dfrac{-1}{2}(z+1)}$ or $\boxed{\dfrac{-(z+1)}{2}}$

numerator

$2z^2+6z+4$

$= 2(z^2+3z+2)$

$= 2(z+2)(z+1)$ (2 × 1 = 2; 2 + 1 = 3)

Note! This is an expression not an equation; must factor out (not remove) GCF.

denom

$-4z-8$ GCF

$= -4(z+2)$

YES (16) $\dfrac{3x^3+3x^2-36x}{6x^3-6x^2-120x}$

$= \dfrac{3x(x+4)(x-3)}{6x(x+4)(x-5)}$

$= \boxed{\dfrac{(x-3)}{2(x-5)}}$

$\frac{x}{x}=1$

$\frac{3}{6}=\frac{1}{2}$

numerator

$3x^3+3x^2-36x$

$3x(x^2+x-12)$ GCF

$3x(x+4)(x-3)$ (−12; 4 × −3 = −12; 4 + −3 = 1)

denom

$6x^3-6x^2-120x$

$6x(x^2-x-20)$ (−20; −5 × +4 = −20; −5 + +4 = −1)

$6x(x-5)(x+4)$

(17) $\dfrac{x^3+4x}{x^4-16}$

$= \dfrac{x(x^2+4)}{(x-2)(x+2)(x^2+4)}$

$= \boxed{\dfrac{x}{(x-2)(x+2)}}$

numerator

x^3+4x

$x(x^2+4)$ sum of squares is prime.

denominator

x^4-16 difference of squares

$= (x^2-4)(x^2+4)$ another diff of sq.

$= (x-2)(x+2)(x^2+4)$

Quick Check 7.1.19

Simplify the rational expression. Assume that variable x has no value which results in the denominator with a value of zero.

$$\frac{15-5x}{5x^2-16x+3}$$

$$\frac{15-5x}{5x^2-16x+3}=\square$$

Step 1: Factor everything completely.

- Numerator: $15-5x$

Standard form $-5x+15$

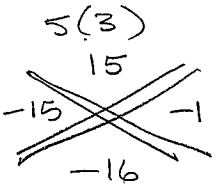
GCF $-5(x-3)$.

- Denominator $5x^2-16x+3$

3 terms, lead ≠ 1 means double x

Rewrite middle term

$5x^2-15x-x+3$



group $5x(x-3)-1(x-3)$

$(x-3)(5x-1)$.

Step 2: Cancel common factors

$$\frac{15-5x}{5x^2-16x+3}=\frac{-5(x-3)}{(x-3)(5x-1)}$$

$$=\boxed{\frac{-5}{5x-1}}$$

7.1.31 Find the value of the variable for which the rational expression is undefined.

$$\frac{6p}{p+5}$$

$p = -5$ (Use a comma to separate answers as needed.)

Set denom = 0

$p+5=0$

solve equation

$\boxed{p=-5}$

7.1.51 Simplify the expression.

$$\frac{x^2-81}{x^2-18x+81}$$

Select the correct choice below and fill in any answer boxes in your choice.

A. $\frac{x^2-81}{x^2-18x+81}=\frac{x+9}{x-9}$ (Simplify your answer.)

B. The expression cannot be simplified.

factor completely

$$\frac{(x-9)(x+9)}{(x-9)(x-9)}$$

$$=\boxed{\frac{x+9}{x-9}}$$

7.1.65 Simplify the rational expression. Assume that variable x has no value which results in the denominator with a value of zero.

① $\dfrac{x^3+7x^2+x+7}{x^2+8x+7}$

$\dfrac{x^3+7x^2+x+7}{x^2+8x+7} = \dfrac{x^2+1}{x+1}$

(Use integers or fractions for any numbers in the expression.)

Standard form ✓

factor completely

$= \dfrac{(x+7)(x^2+1)}{(x+7)(x+1)}$

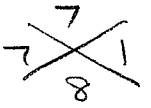
① x^3+7x^2+x+7

$= x^2(x+7)+1(x+7)$

$= (x+7)(x^2+1)$.

② x^2+8x+7

$=(x+7)(x+1)$



Cancel common factors

$\dfrac{(x+7)}{(x+7)}$

$= \boxed{\dfrac{x^2+1}{x+1}}$

Note: x^2+1 is a sum of squares, which is prime and cannot be factored.

7.1.67 Simplify the rational expression. Assume that no variable has a value which results in a denominator with a value of zero.

$$\frac{9-t^2}{(t-3)^2}$$

$$\frac{9-t^2}{(t-3)^2} = \boxed{-1}$$

standard form.

① $-t^2+9$

② $(t-3)^2$

factor completely

$$= \frac{-(t-3)(t+3)}{(t-3)(t-3)}$$

① $-t^2+9$

$= -(t^2-9)$

$= -(t-3)(t+3)$

② $(t-3)^2$

already factored

$(t-3)(t-3)$

cancel common factors

$$\frac{t-3}{t-3}$$

$$= \boxed{\frac{-(t+3)}{(t-3)}}$$

7.1.71 Simplify the rational expression. Assume that no variable has a value which results in a denominator with a value of zero.

$$\frac{18-7x-x^2}{x^2-81}$$

$$\frac{18-7x-x^2}{x^2-81} = -\frac{x-2}{x-9}$$

YOU ANSWERED: $-\frac{2+x}{x+9}$ check dist neg $\Rightarrow$ $\frac{-2-x}{x+9}$

write in standard form:

$$\frac{-x^2-7x+18}{x^2-81}$$

factor completely

$$\frac{-1(x^2+7x-18)}{(x-9)(x+9)}$$

-18: 9×-2, $9 + (-2) = 7$

$$\frac{-1(x+9)(x-2)}{(x-9)(x+9)}$$

$$= \boxed{\frac{-(x-2)}{(x-9)}}$$

check dist neg $\Rightarrow$ $\frac{-x+2}{x-9} = \frac{2-x}{x-9}$

Extras

Simplify

(18) $\dfrac{x^2-9}{2x^2-3x-9}$

(19) $\dfrac{456}{420}$

(20) $\dfrac{45a^2b^3}{15ab}$

(21) $\dfrac{7x+14}{x^2-4}$

(22) $\dfrac{a^2+3a-28}{2a^2-a-28}$

(23) $\dfrac{12-4x}{4x^2-13x+3}$

(24) $\dfrac{12w-3w^2}{w^3-5w^2+4w}$

(25) $\dfrac{x-3}{x^2-4x+3}$

(26) $\dfrac{2v^2-6v}{3-v}$

(27) $\dfrac{2k^2-14k}{7-k}$

(28) $\dfrac{5m+5n}{m^2+2mn+n^2}$

(29) $\dfrac{9-x^2}{(x-3)^3}$

(30) $\dfrac{5+4x-x^2}{x^2-25}$

Find values of variable which make expression undefined.

(31) $\dfrac{3h+2}{h^3+5h^2+4h}$

(32) $\dfrac{12x+5}{x^3-x^2-6x}$

(33) $\dfrac{2x^2}{x^2+x-2}$